Homework 3. (Week 4)

In class we found that the natural decay frequency of an underdamped oscillator is $w = \sqrt{w_0^2 - (1/2\tau)^2}$.

Does this correspond to the resonant frequency of the same system? Let's solve the driven LRC circuit for $v(t)$ in steady state.

$$\frac{d^2v}{dt^2} + \frac{1}{\tau}\frac{dv}{dt} + w_0^2 v = X_0 \sin wt$$

Hints: (1) Use $v(t) = V_0 \sin(wt+\phi)$ as a trial function.

(2) Recall the expensions for $\cos(a+b)$ and $\sin(a+b)$.

(3) Group terms proportional to $\sin wt$ and $\cos wt$.

(4) The final result should be in the form

$$V(t) = \frac{X_0}{\text{function}(w, w_0, \tau)} \sin[wt+\phi]$$

Minimize the function of w, w_0, and τ. to find the resonant frequency.

Plot the amplitude and phase of $v(t)$ versus frequency as a Bode plot

State a formula for $w_{resonance} = \text{function}(w_0, \tau)$